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# SLIP-STREAM CORRECTIONS IN PERFORMANCE COMPUTATION

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## SLIP-STREAM CORRECTIONS IN PERFORMANCE COMPUTATION

BY

EDWARD P. WARNER

144538—20——1

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## SLIP-STREAM CORRECTIONS IN PERFORMANCE COMPUTATION.

By Edward P. Warner.

In computing the performance of an airplane, the values taken for the slip-stream correction have a very marked effect on the results, both as regards the speed required for flight at a given angle of attack and as regards the horsepower required, and a mistake in estimating the magnitude of the correction factor may cause considerable errors in the preliminary estimate of performance attainable. Notwithstanding the importance of the slip-stream effect relatively little study has been devoted to it and the proper way of treating it still remains largely uncertain. This lack of knowledge as to the magnitude of the correction and its effects is amply attested by the wide variation among the methods employed by designers engaged in the making of performance computations. Some have ignored the correction entirely. Others, seeking a slightly greater accuracy but not desiring to embark on too laborious a set of computations, have made an over-all allowance, increasing the total resistance of the machine (including both parasite resistance and wing drag) by 10 per cent or 15 per cent. Such a device is frequently employed when performance is to be computed directly from a wind-tunnel test on a model of the complete machine, and when only the total resistance is known, no data on the resistance contributed by different parts of the structure being available. As the proportion of the total resistance which lies in the slip stream varies widely between different types of machines, this method can manifestly lead to nothing more than a very rough approximation. The next step, to further increase the exactness of the performance computations, is to consider the resistance broken up into two parts; that due to members inside and that due to members outside the slip stream, and to treat these two parts as having different relative air speeds. The speed used in computing the resistance of the parts outside the slip stream area is equal to the air speed of the machine; that used for the parts inside the slip-stream is somewhat higher. The amount of the difference between the two has been much in dispute, but the corrections generally applied range from 10 per cent to 20 per cent. That is to say, the velocity is considered to be from 10 per cent to 20 per cent greater inside than outside the slip stream, and the resistance of an object in that area is therefore taken as from 21 per cent to 44 per cent greater than the resistance of the same object would be if it were removed from the zone of influence of the propeller draft.

Whatever the value that may be taken for the correction factor, it is almost invariably considered to remain constant at all speeds of flight, and it is in this assumption that the greatest source of error lies. When the engine is kept running at full throttle, and all the reserve power is used in causing the machine to climb, the air speed being relatively low, the ratio of slip-stream velocity to velocity of advance will quite evidently be considerably higher than when the flight path is horizontal, with a higher speed of advance or with a throttled engine. The exact manner of the variation of slip-stream effect will be taken up a little later.

We may attack the problem of the slip-stream correction in either of two ways, both of which we shall discuss. In the first place, we may base our corrections on wind-tunnel tests of propeller models. Unfortunately, there is very little data available on the velocity of the flow of the air behind a propeller under test. Secondly, we may depend on pure theory, determining the mean slip-stream velocity from energy considerations. While this is not strictly valid, it affords an interesting means of obtaining data for comparison, and of extending the experimental results.

144538—20——2

## EXPERIMENTAL RESULTS.

Measurements of slip-stream effect for model propellers have been made by Eiffel, at his laboratory near Paris, and by Riabouchinsky at Koutchino. Eiffel's results are the more valuable, as the propeller used during the tests was more nearly like those used on aircraft at the present time than were the models employed at Koutchino.

It should be noted that all these experiments deal with propellers tested with no other objects in their neighborhood, and that the results might be materially modified by the mutual interference of the propeller and the other parts of the machine. This is particularly true of the body, inasmuch as it is very close to the propeller and is immediately behind the least efficient portion of that member, so that it has somewhat the same effect of stream-lining the hub and guiding the air across the more effective portions of the blade as has a spinner in front. The effect of the body on the propeller would then be such as to neutralize, at least in part, the increase of body resistance caused by the slip-stream. This effect would apply primarily to the body, the resistance of which, if it is well designed, is a relatively small part of the total parasite resistance in the slip-stream. The effect of the struts, tail surfaces, and other members on the action of the propeller would probably be so small as to be negligible, thanks to their distance behind the propeller and to the fact that they lie behind the effective portion of the blades rather than directly behind the hub.

The ratio of slip-stream velocity to velocity of advance, like every other factor or propeller performance, is, for geometrically similar propellers, a function of the nondimensional ratio $V/ND$. Since it is necessary to apply to widely different propellers the data obtained by tests of two or three special cases we must seek some means of comparison, and this means is provided by the provisional assumption that the ratio of slip-stream velocity to flight velocity at the speed of maximum propeller efficiency is the same for all propellers. We may then consider the slip-stream effect to depend on the ratio of $V/ND$ to $(V/ND)'$, or, for a given propeller diameter and engine speed, on $V/V'$, where $V$ is the actual speed of flight and $V'$ the speed for maximum propeller efficiency at the same number of revolutions per minute. This is the only method open to us if we rely solely on experimental data, but we shall see later that the use of the momentum theory makes it possible to actually compute the slip-stream effect for a given propeller, and that the magnitude of this effect varies somewhat with blade form, blade width, number of blades, pitch-diameter ratio, etc.

Eiffel's experiments [1] were performed on his propeller No. 9, which has quite a normal blade form, with straight trailing edge and leading edge so curved as to give a maximum blade width approximately three-quarters of the way out from hub to tip. The pitch at all points was 0.7 of the diameter, and the maximum efficiency (75 per cent) was secured when $V/ND$ was equal to 0.6, corresponding to a speed of flight of roughly 105 miles per hour with a Liberty engine turning 1,700 revolutions per minute and a two-bladed propeller 9 feet in diameter. The ratio of effective pitch to diameter for best efficiency therefore was not very far from current practice.

The air velocity in the slip-stream was determined by pitot tube measurements at five points, placed at 0.4, 0.6, 0.8, 0.96, and 1.1 times the radius from hub to tip. That is, the last point was actually slightly outside of the propeller disk area. To determine the extent to which the air spreads out and the added velocity disappears as the distance from the propeller is increased, three such sets of points were tried, the first being only 0.04 of the propeller diameter behind the plane of the trailing edges of the propeller blades, the second being located 0.2 of the diameter behind that plane, and the third being separated from the propeller by a distance equal to its diameter.

The conclusions to be drawn directly from the results of these tests may be tabulated briefly as follows:

(*a*) The slip-stream velocity is a maximum at a radius of about 0.6 of the distance from hub to tip.

[1] "Nouvelles Recherches sur la Resistance de l'Air et l'Aviation," by G. Eiffel: Paris, 1914; p. 333.

(b) At low speeds of advance and large slip percentages the slip-stream velocity drops off very rapidly near the edges of the propeller disk. In the case under examination, at moderate translational speeds ($V/V'=0.4$ to $V/V'=0.6$), the added slip-stream velocity (i. e., the difference between $Vs$ and $V$) at the radius 0.96 was from 40 per cent to 70 per cent less than that at radius 0.8. At higher speeds of advance, a decrease in slip-stream in going out from one of these points to the other is still noticeable, but it is much less abrupt and less marked.

(c) At points which are neither very near to the center nor to the edge of the propeller disk, the velocity varies very little with distance from the propeller, at least within the limits of these experiments. At the higher speeds of translation, there is a slight tendency for the velocity to increase as the distance from the propeller increases.

(d) Near the center of the propeller the velocity drops off somewhat, but not nearly so much or so abruptly as it does near the tip.

(e) At low velocities of advance, the velocity near the center of the propeller disk increases considerably as the distance from the propeller increases.

(f) Near the edge of the slip-stream, the velocity is greatest very close behind the propeller. On going back from the propeller the velocity decreases quite rapidly for a short distance, and then, on going still farther away, shows a tendency to increase slightly. (e) and (f), taken together, indicate that the slip-stream diameter is nearly independent of distance from the propeller, but that it contracts rapidly for a short distance passing through the propeller. The point of minimum section having been reached, the slip-stream begins to expand in cross-section, but only very slowly. We shall not go far astray if we consider the slip-stream velocity and section to be independent of the distance from the propeller.

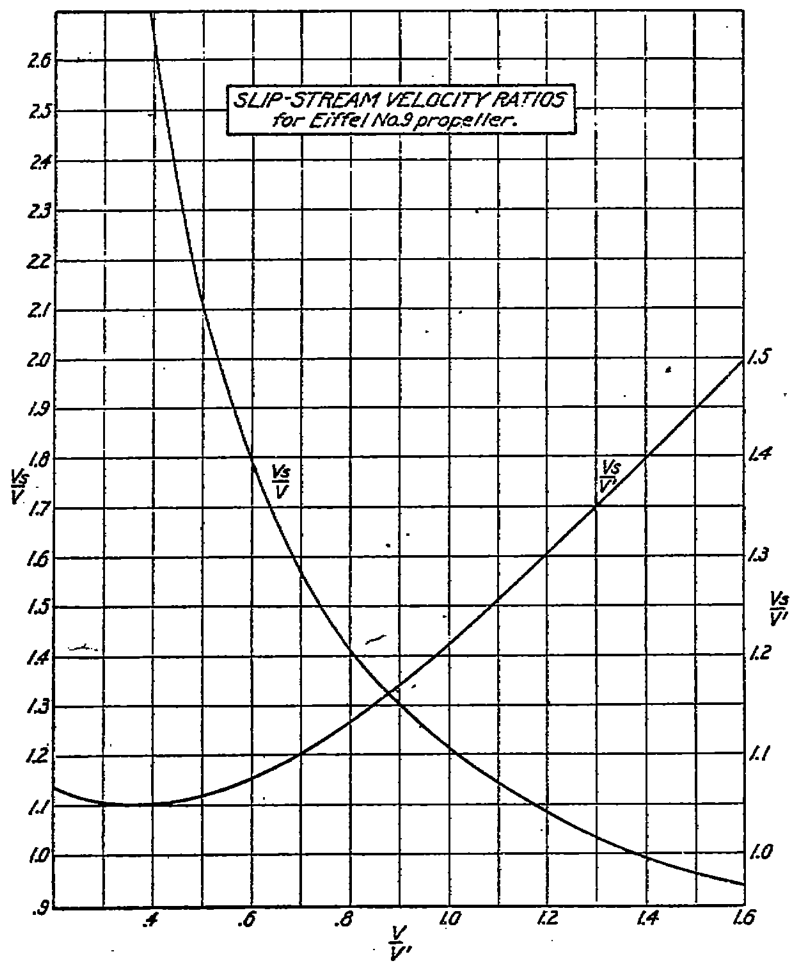

FIG. 1.

Summing up, it appears that we shall secure reasonable results if we take the slip stream as having 0.9 the diameter of the propeller and assume the velocity to be constant over the stream so defined. Mean values, obtained in this way, of $V_s/V$ and $V_s/V'$ have been plotted against $V/V'$ in figure 1, $V$ and $V'$ being the actual velocity of advance and the velocity of advance for best propeller efficiency, as before, $V_s$ being the velocity in the slip stream. $N$ and $D$ are assumed to remain constant throughout. Any change in these quantities will necessitate a recomputation of $V'$.

Although the above assumption as to slip-stream diameter may be accepted as registering sufficiently closely with experimental results, it can be shown theoretically that the diameter of the stream should vary with slip ratio, or, for a given engine speed and propeller diameter, with the speed of flight. R. E. Froude has shown that the increase in velocity of the fluid before it reaches the propeller should be exactly equal to the added velocity imparted after passing through the propeller, or, using algebraic symbols, that $V_1 - V = \frac{1}{2}(V_s - V)$, where $V$ is the speed of flight, $V_1$ the indraught velocity, and $V_s$ the slip-stream velocity. Since the same mass of fluid passes every point along the stream in a unit of time, the product of velocity and stream area must be the same at all points if compressibility of the fluid be neglected. We shall then have, assuming the cross-section area of the inflowing current to be equal to the area swept by the propeller:

$$A_s \times V_s = \frac{\pi D^2_s}{4} \times V_s = \frac{\pi D^2}{4} \times V_1 = \frac{\pi D^2}{8}\ (V_s + V)$$

$$D_s^2 \times V_s = \frac{D^2}{2}\ (V_s + V)$$

$$\frac{D_s}{D} = .71\sqrt{\frac{V_s + V}{V_s}} = .71\sqrt{1 + \frac{V}{V_s}}$$

$V_s = 1.645V$

This ratio would vary from a minimum value of 0.71, under static conditions, to a maximum of 1.0 at the speed where $V_s = V$ and there is no thrust. As neither of these conditions is ever approached in normal flight, we can safely say that the theoretical value of $\frac{D_s}{D}$ will not wander far from the one (0.9) to which we have already been led by empirical data.

The most striking thing about the curves of figure 1 is the remarkable degree of constancy of $V_s/V'$. It appears that, so long as the engine speed is kept constant, the slip-stream velocity is almost independent of the speed of flight, except at speeds well above that of maximum efficiency.

Since $V_s$ is so nearly constant, it is evident that $V_s/V$ will increase rapidly as $V$ decreases, becoming infinite when the machine is being held stationary on the ground with the engine running, prior to the starting of a flight. The method, which I have already mentioned, of computing performance by assuming $V_s/V$ to have a constant value of 1.15 or 1.2 is therefore manifestly quite incorrect except for a single speed somewhere in the neighborhood of the speed of maximum propeller efficiency. To determine the correction factor for any other speed we must have recourse to slip-stream velocity curves similar to those of figure 1. The error introduced by the assumption of a constant correction factor will obviously be most important at the low and moderate speeds which correspond to most efficient climb, and undoubtedly accounts for the difficulty which has usually been experienced in computing rate of climb. It is well known that, although the maximum horizontal speed of a new machine can be predicted with a high degree of accuracy from performance computations of the ordinary type or from a wind-tunnel test of a model, an attempt to predict climbing speed in a similar fashion very commonly gives a value distinctly higher than that found in a free-flight test of the completed machine.

## EFFECT OF SLIP-STREAM ON PERFORMANCE.

The method of applying the slip-stream correction to the parasite resistance of the parts inside the slip-stream area to secure the corrected total resistance and the horsepower required for flight is simple and obvious, differing in no essential particular from that employed when the correction factor is assumed to remain constant.

We have been considering so far a case in which the engine speed is assumed to remain constant at all speeds of flight, a case which is never exactly realized. If the throttle be kept open, the load on the engine will be greatest, and the revolutions per minute will consequently be lowest, when the slip percentage is greatest; or, in other words, when the speed of flight is lowest, and the engine speed with open throttle may be 5 per cent or more lower when the plane is climbing at an angle of attack of about 8° than when it is flying level at an angle of attack in the neighborhood of $-1°$. The variation of revolutions per minute is small, however, and we shall not go far astray if we stick to our original assumption that the engine speed remains constant so long as the throttle is wide open. We have now to consider the effects of partially throttling the engine, and the modifications of slip-stream effect which are involved. When the throttle is partially closed the engine speed is reduced and the value of $V/ND$, the propeller slip function, is therefore increased. It is evident from Fig. 1 that an increase in $V/ND$ causes a decrease in the ratio of $V_s$ to $V$. If the engine be throttled to half its original number of revolutions, for example, maintaining the same speed of flight and varying the slope of the flight-path, the value of $(V/ND)/(V/ND)'$ will be doubled and the ratio of $V_s$ to $V$ will be reduced, for a machine flying at 40 per cent of the speed for which the propeller was designed, from about 2.6 to 1.4. The parasite resistance of parts in the slip-stream would then be reduced by 71 per cent by throttling the engine, although the speed of flight would remain the same. This, of course, is an extreme case, as the engine speed in level flight would seldom drop to less than 75 per cent of the rated speed. If the plane is gliding with the engine cut off, the slip-stream effect will of course disappear entirely.

It is of interest now to examine the variation of the slip function and of the slip-stream correction when the machine flies at different speeds in the neighborhood of the maximum attainable, the engine always being throttled just sufficiently to keep the flight-path horizontal. When the plane is flying at high speed the angle of attack is small and the coefficient of wing drag is near its minimum value. This coefficient $D_0$ will therefore remain almost constant if the angle is varied slightly, and the total wing drag will be very nearly proportional to the square of the speed for small changes in speed in the neighborhood of the maximum attainable. The parasite resistance is always proportional to the square of the speed, neglecting the small effect of inclination of the body, struts, etc., and the total resistance will therefore be proportional to the square, and the horsepower required to the cube, of the speed of flight, provided that the slip-stream correction factor remains constant. This relation, it must be remembered, holds only in the neighborhood of the maximum speed of flight (say for angles of attack between $-2°$ and $+3°$). It can be shown that, if we assume the validity of the blade element theory of propeller design, the power absorbed by a given propeller is proportional to $V^3$ if the slip function be kept constant. Since the horsepower absorbed must vary as $V$ in order that it may be equal to the horsepower required for level flight, it is evident that the slip function will be approximately constant so long as the machine flies in a horizontal path at an angle close to that of minimum drag. To take a concrete illustration, if the speed of flight be decreased 10 per cent the maximum power required for flight will decrease to $(.9)^3$, or 73 per cent of the original value. In order that the slip function may remain constant the engine speed must be decreased by 10 per cent. Since the value of $V/ND$ is unchanged, the ratio of slip-stream velocity to speed of flight will also remain unchanged, so long as the flight-path is horizontal and the speed is fairly high. At very low speeds the slip-stream correction factor in horizontal flight is larger than at those in the neighborhood of the maximum, as the revolutions per minute decrease less rapidly than the flight speed, and the slip function consequently decreases at low speeds.

## DEDUCTIONS FROM THE MOMENTUM THEORY OF PROPULSION.

Our deductions to date have all been based, owing to an unfortunate paucity of experimental data, on a single set of tests in which only one propeller was used. It is now of interest to examine the question from another point of view, and see what we can learn by the application of pure theory.

The Rankine-Froude theory of fluid propulsion is based on the assumption that the thrust given by a propeller is equal to the sternward momentum imparted to the fluid in unit time.

If we assume the correctness of this theory, and further assume, as we have already done, that the slip-stream has a diameter equal to 0.9 of the propeller diameter and that the velocity of the air is uniform all over the slip-stream, we can readily compute the slip-stream velocity, given the engine horsepower, propeller efficiency, propeller diameter, and speed of flight.

Let $P$ = horsepower.

$T$ = propeller thrust.

$\eta$ = propeller efficiency.

$D$ = propeller diameter (feet).

$V$ = speed of flight (feet per second).

$V_s$ = slip-stream velocity (feet per second).

$A_s$ = area of slip-stream $= .81\frac{\pi}{4}D^2 = .636\ D^2$.

$\rho$ = density of air = .07608 pounds per cubic foot under standard conditions.

$M$ = mass of air passing through propeller in 1 second $=\frac{\rho}{g}\times V_s\times A_s$.

$$P=\frac{T\times V}{550\times\eta}$$

$$T=\frac{550\times P\times\eta}{V} \tag{1}$$

$2(V_1 - V)$

From the momentum theory of fluid propulsion,

$$T=M\times(V_s-V)=\frac{\rho}{g}\times A_s\times V_s\times(V_s-V)=.0015\ D^2\times V_s\times(V_s-V) \tag{2}$$

Equating (1) and (2),

$$\frac{550\times P\times\eta}{V}=.0015\ D^2\times V_s\times(V_s-V)$$

$$(V_s-V)\times V_s=\frac{367{,}000\times P\times\eta}{V\times D^2}$$

It is a well-known fact that, for any given value of the slip function, $V/ND$, the power consumed in driving a propeller of any particular type is proportional to $V^3D^2$. We can then write $K_1 V^3D^2$ in place of $P$ in the above equation.

$$(V_s-V)\times V_s=367{,}000\ K_1\ V^2\eta$$

Dividing through by $V^2$

$$\left(\frac{V_s}{V}-1\right)\times\frac{V_s}{V}=367{,}000\ K_1\times\eta$$

and solving:

$$\frac{V_s}{V}=\frac{1}{2}\left(1+\sqrt{1+1{,}468{,}000\ K_1\eta}\right) \tag{3}$$

It is evident that any change in propeller design, such as an increase in the number of blades which tends to increase the power absorbed by a propeller of given diameter, will increase the magnitude of the slip-stream correction as given by this formula.

It is of considerable interest to compare the slip-stream correction obtained by this theoretical analysis with that found by actual measurement of the air velocities behind a propeller. This has been done for Eiffel's propeller No. 9, the experimental data for which we have already studied. A table of $K_1$, $\eta$, and the theoretical slip-stream correction for various values of $V/ND$ is given below, and figure 2 shows the comparison between the theoretical and experimental values of $V_s/V$.

| $V/ND$ | $K_1 \times 10^6$ | $\eta$ | $V_s/V$ |
|---|---|---|---|
| 0.347 | 6.02 | 0.690 | 1.745 |
| .433 | 2.91 | .672 | 1.484 |
| .503 | 1.73 | .725 | 1.341 |
| .540 | 1.35 | .742 | 1.285 |
| .614 | .76 | .750 | 1.178 |
| .694 | .52 | .710 | 1.120 |
| .754 | .31 | .640 | 1.067 |
| .812 | .17 | .390 | 1.024 |
| .857 | .14 | .048 | 1.002 |

The coincidence between the two curves in figure 2 is quite extraordinary, especially in the neighborhood of the point of best propeller efficiency. At no point in the range corresponding to conditions of normal flight do the two values disagree by more than 2 per cent. This exactitude of agreement must, however, be regarded as largely fortuitous, as Eiffel's experiments themselves could hardly be accurate to such a degree.

The slope of the theoretical curve is a little less than that of the experimental one, but even this slight discrepancy can be accounted for by our failure to take account of the variations in diameter of the slip stream, which, as we have already seen, varies somewhat in size with changing speed of flight.

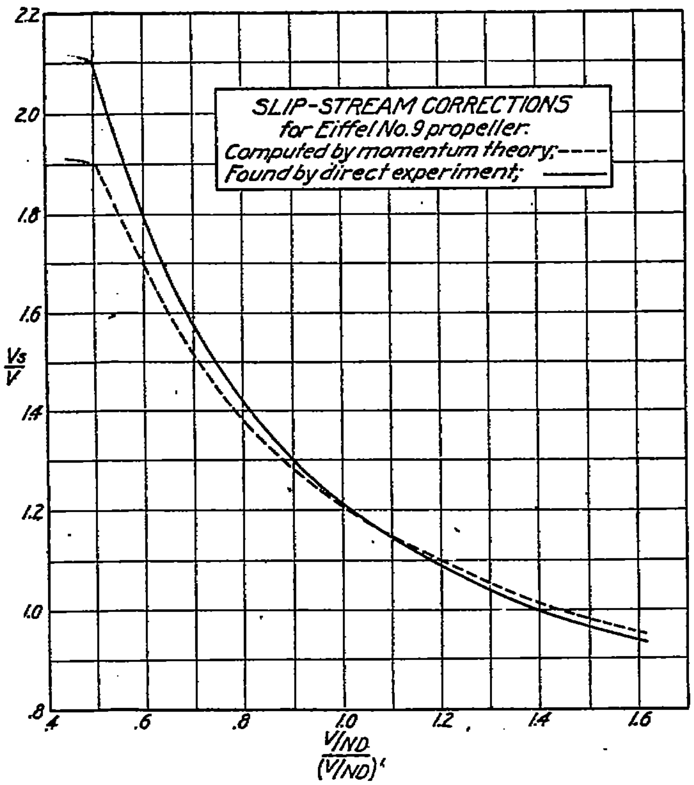


FIG. 2.

It appears that the formula (3) can be used without hesitation to secure the slip-stream correction and the correction can undoubtedly be determined with much greater accuracy in this way than by any adaptation of the results of experiments on propellers of type differing from that which is to be employed. $K_1$ can always be determined for any machine when the speed of flight, propeller diameter, and horsepower delivered by the engine under any given conditions are known. The efficiency can be computed from the propeller drawings by the standard blade element method. The application of (3) is, however, somewhat more tedious than is the simple process of reading the slip-stream correction from the curve of figure 1, and the choice between the two methods, where the propeller is close to the usual form, is largely a matter of personal preference, with the computation by the momentum theory having the advantage in respect to accuracy.

In order to check the validity of our original assumption that the slip-stream correction factors at the point of maximum efficiency is the same for all propellers, the values of this factor have been computed, by formula (3), for all the propellers tested by Eiffel up to 1914, and also for some of those tested by Dr. Durand.[1] The values of the correction for the various propellers are tabulated below:

*Eiffel.*

| No. | $V_s/V$ |
|---|---|
| 1 | 1.354 |
| 2 | 1.245 |
| 3 | 1.122 |
| 4 | 1.161 |
| 5 | 1.435 |
| 6 | 1.361 |
| 7 | 1.235 |
| 8 | 1.170 |
| 9 | 1.178 |
| 10 | 1.155 |
| 11 | 1.165 |
| 12 | 1.140 |
| 13 | 1.241 |
| 14 | 1.263 |
| 15 | 1.219 |
| 15a | 1.258 |
| 16 | 1.373 |
| 17 | 1.373 |
| 18 | 1.439 |
| 19 | 1.095 |
| 20 | 1.065 |
| 21 | 1.159 |
| 22 | 1.339 |
| 23 | 1.170 |
| 24 | 1.108 |
| 25 | 1.166 |
| 26 | 1.233 |

*Durand.*

| No. | $V_s/V$ |
|---|---|
| 1 | 1.169 |
| 2 | 1.178 |
| 3 | 1.126 |
| 4 | 1.145 |
| 5 | 1.109 |
| 6 | 1.245 |
| 7 | 1.205 |
| 8 | 1.241 |
| 9 | 1.285 |
| 10 | 1.330 |
| 11 | 1.330 |
| 12 | 1.329 |
| 13 | 1.151 |
| 17 | 1.222 |
| 21 | 1.296 |
| 25 | 1.183 |
| 29 | 1.271 |
| 33 | 1.390 |

The variation in $V_s/V$ is greater than might have been expected, Eiffel's 27 propellers showing values ranging from 1.065 to 1.439. Just above half of the models tested have values between 1.15 and 1.35, and we can safely say that these will be the limiting values for two-bladed propellers of normal type and pitch. In the case of the Durand propellers, $V_s/V$ lay between 1.15 and 1.35 for all except 2 of the 18 examined.

The figures obtained above lead to certain general conclusions as to the dependence of the slip-stream effect on the type of propeller employed. It appears that $V_s/V$ depends primarily on pitch, being high for those propellers in which the pitch-diameter ratio is least. Eiffel's No. 1, for example, which shows a very high $V_s/V$, had a pitch of only about one-half the diameter, whereas No. 20, for which $V_s/V$ was only 1.065, had a pitch of 1.4 times the diameter. It appears, furthermore, and rather surprisingly, that the slip-stream effect is substantially independent of blade width and blade form, the propellers with blades of constant width showing, on the whole, a slightly higher $V_s/V$ than those with more rounded blade tips.

The use of a cambered face on the blades increases $V_s/V$. Durand Nos. 5 and 29, for example, are exactly alike except that the former has a flat, the latter a cambered, blade face, and No. 29 gives a considerably higher slip-stream velocity than does No. 5. The most pronounced effect, however, comes from varying the number of blades. Eiffel's Nos. 15 and 16 are exactly similar except that the former has two blades, the latter four (Nos. 17 and 18 are also four-bladed), yet the first gives a velocity ratio of 1.22, the second of 1.37. It is customary, in using the Drzewiecki, or blade element, theory of propeller design, to assume that two narrow blades are exactly the same as one wide one, but analysis of experiments makes it appear that that assumption is far from the truth, and that a given blade area will absorb more power when it is subdivided among several blades than when it is concentrated in two.

[1] Third Annual Report of National Advisory Committee for Aeronautics; Washington, 1918.